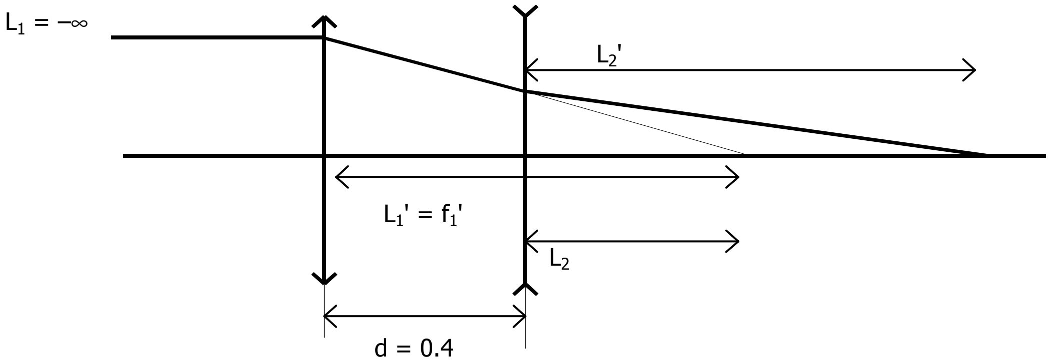
# Optics 8 : Thin lenses in combination

Problems involving thin lenses in combination can be solved by successive application of the thin lens formulae

**For example:** 2 thin lenses with powers of +2 dioptres and -5.00 dioptres are separated by 40cms. Find the image position for a distant object:



First method - use thin lens formula, and calculate position of the image formed by the first lens in isolation, consider this as the object for the second lens, and then calculate final image position:

**For first lens**:
focal length = $f_1'$ = 1/power of lens = 1/2 = 0.50 m

Light striking the lens from infinity will be directed towards the second focal point of the lens (ie +0.50m from the lens). Using the Gaussian lens formula

$$\frac{1}{L_1'}-\frac{1}{L_1}=\frac{1}{f_1'} \qquad \text{but since } L_1=-\infty \quad \Rightarrow \quad 1/L_1=0$$

$$\Rightarrow \quad \frac{1}{L_1'}=\frac{1}{f_1'} \qquad \text{and} \qquad L_1' = f' = 0.50 \text{ m.}$$

**For the second lens:**
focal length = $f_2'$ = 1/power of lens = 1/(-5) = –0.20 m

Object distance for lens 2: the image formed by the first lens acts as the object for the second lens. The image formed by the first lens would be $L_1$ = 0.50m from the first lens, and the second lens is $d$ = 0.40m from the first lens. Thus, the image formed by the first lens is $L_1 - d$ = 0.10 m to the right of the second lens.
In our sign convention +0.10 m from the second lens

$$\therefore L_2 = +0.10$$

Using the Gaussian lens formula again

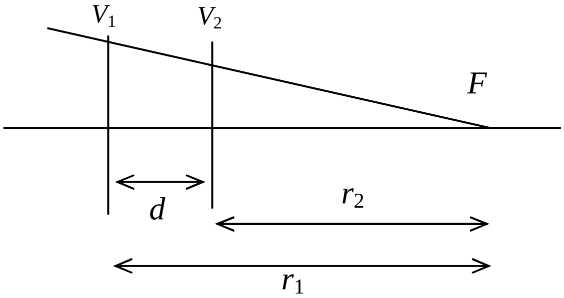
$$\frac{1}{L_2'}-\frac{1}{L_2}=\frac{1}{f_2'}$$

$$\frac{1}{L_2'} - \frac{1}{0.1} = \frac{1}{-0.2} \quad \text{giving} \quad \frac{1}{L_2'} = +\frac{1}{0.1} - \frac{1}{0.2} \quad \text{or}$$

$$\frac{1}{L'} = +5.0 \quad \text{or} \quad L_2' = 0.20 \text{ m}$$

So the image appears 0.20 m to the right of the second lens.

<u>Second method</u> – convert distances to vergences – but first we need to find how vergence changes over distance. Suppose that rays converge on a point F.

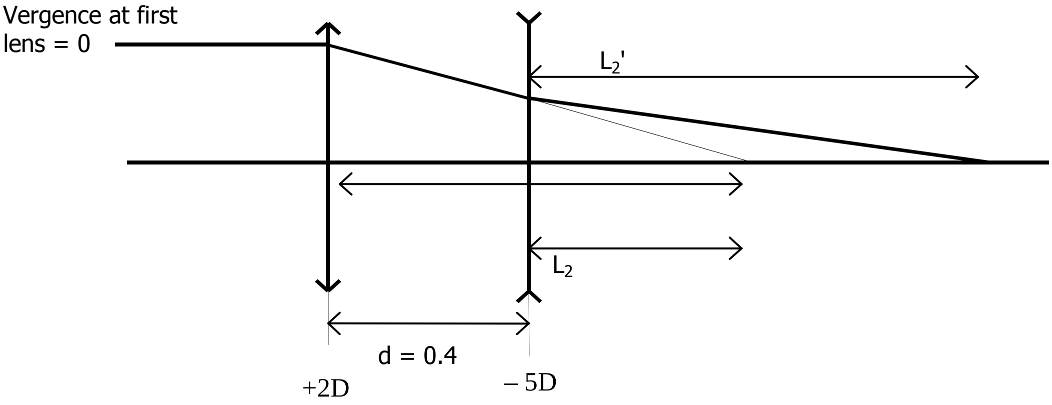


The vergences at $r_1$ and $r_2$ are given by

$$V_1 = \frac{1}{r_1} \quad \text{and} \quad V_2 = \frac{1}{r_2}$$

but $r_2 = r_1 - d$ Hence $V_2 = \dfrac{1}{r_1 - d} = 1\Big/\left[\dfrac{1}{V_1} - d\right] = \dfrac{V_1}{1 - V_1 d}$

Now returning to the example

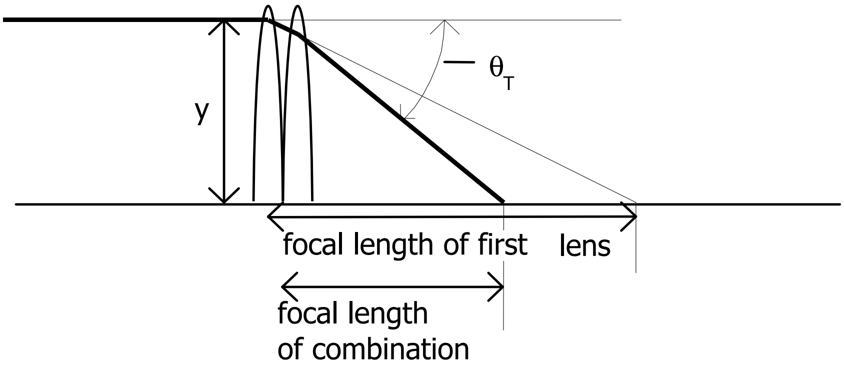


Vergence of light as it strikes first lens = 0.
Vergence of light as it leaves first lens $V_1 = +2$D.
Vergence of light after converging over distance $d$ $V_2 = 2/(1 - 2\times 0.4) = +10$D
Vergence of light as it leaves second lens +10D – 5D = +5D
Image is therefore +5D from second lens ≡ 0.2 m.

## Two thin lenses in contact

If, in the above example d -> 0, so that the two thin lenses are in contact we find that the power of the combination is just the sum of the powers of each lens.

This can also be shown as follows:



for two thin lenses with powers $P_1 = 1/f_1'$ and $P_2 = 1/f_2'$ that deviate an incident ray by $\theta_1$ and $\theta_2$ respectively. In the Paraxial approximation: $\theta_T \approx \theta_1 + \theta_2$ and

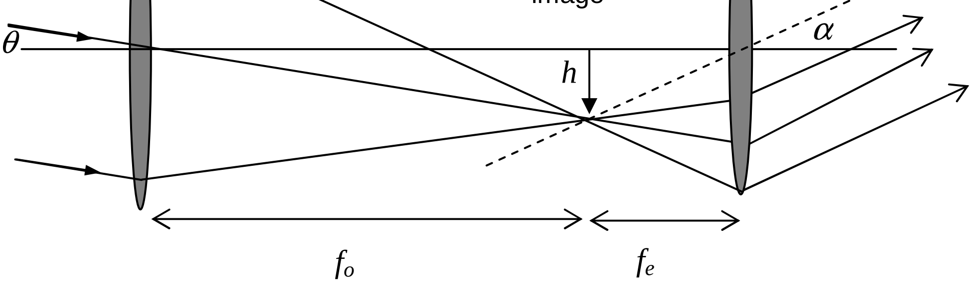
$$\Rightarrow \; y/f_T' = y/f_1' + y/f_2'$$
$$1/f_T' = 1/f_1' + 1/f_2'$$

Hence the total power, $P_T$ is given by

$$\therefore \quad P_T = P_1 + P_2$$

## Astronomical Telescope

The astronomical telescope is a combination of two converging lenses the objective and the eyepiece



Parallel rays from a distant object enter the objective at an angle $\theta$ and form a real intermediate image at the second focus of the objective focal length $f_o$. The image forms the object for the eyepiece. If the image is placed at the first focus of the eyepiece the emerging rays will also be parallel but the angle $\alpha$ subtended by the image is increased and inverted. Since $\tan\alpha = h/f_e$ and $\tan\theta = h/f_o$ and both $\alpha$ and $\theta$ are small

$$M_T = -\frac{\alpha}{\theta} = -\frac{f_o}{f_e}$$

Astronomical telescopes need large objectives to gather light from dim objects. Terrestrial telescopes include an extra lens to give an erect image.

## Chromatic aberration

Prisms disperse light because the refractive index changes as the wavelength of light alters. Similarly lens power depends on refractive index, and since this is different for different wavelengths, lens power depends on the wavelength used. This is *chromatic aberration* and appears as coloured blurring of images. It can be corrected by combining together two lenses made of different glass with different dispersions in an *achromatic doublet.*