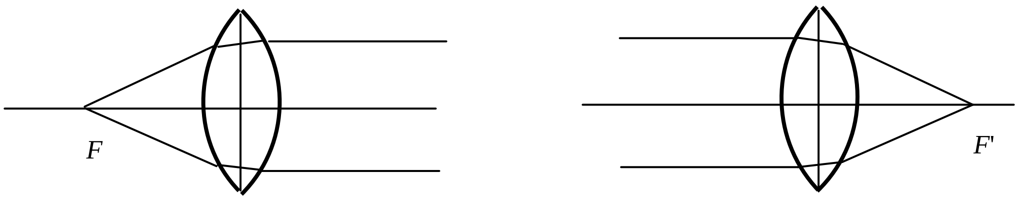
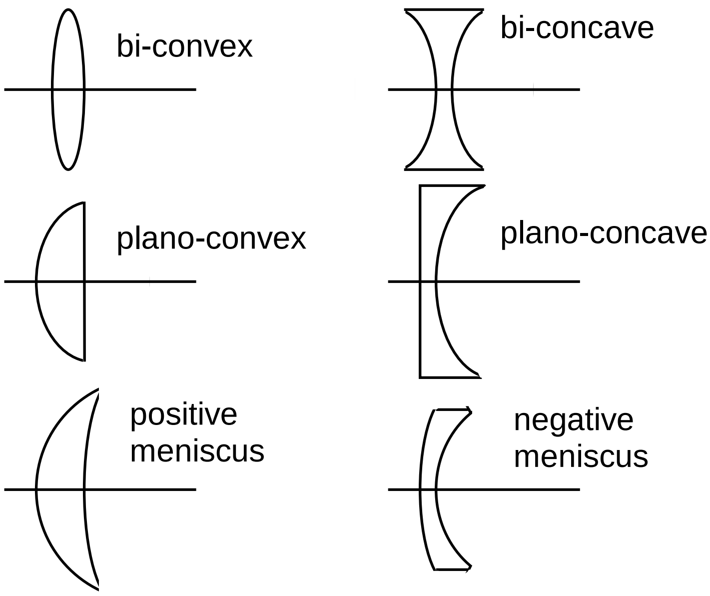
## Optics 7:   Spherical Lenses

A lens could be defined as a portion of transparent substance bounded by two polished surfaces, one or both of which is curved. A converging lens could be made of two spherical convex surfaces:




Rays from a point source at the first focal point, F, converge and emerge parallel to the axis.  Rays parallel to the axis are focused at the second focal point F' .  Similarly a diverging lens could be made from two concave surfaces. The surfaces, the refractive index of the lens, and its thickness determine the effect which that lens has on light.




Convex lenses are commonly termed *convergent* or *positive* lenses, and concave lenses are likewise termed as *divergent* or *negative* lenses. This is because convex lenses converge rays of light and have a positive effect on the vergence of wavefronts and concave lenses diverge rays and have a negative effect on vergence.

### Thin Lenses

A *thin* lens is an idealized model where the thickness of the lens is small compared to other distances involved (e.g. the focal length, radii of curvature, object and image distance etc.), and we can therefore ignore the effect of the thickness of the lens. This simplifies any calculations involved, but is an approximation. Thick lenses (where the effect of the substance of the lens cannot be ignored) will be considered later.  Also, a lens shows its most ideal behaviour towards light rays passing it close to its center (paraxial) and nearly parallel to its optic axis. In the eye, the pupil (at least in good lighting) only lets through centrally located rays, and a good image is only needed at the fovea centralis. Therefore, ideal lens optics are useful in the eye, in spite of the very strong curvature of the refracting surfaces.

By combining two spherical refracting surfaces we enclose a lens. Similarly, by combining the mathematical descriptions :

$\frac{n_i}{x_o}-\frac{n_t}{x_t}=\frac{n_i-n_t}{r_1}$ for the first surface and $\frac{n_t}{x_t}-\frac{n_i}{x_i}=\frac{n_t-n_i}{r_2}$ for the second we obtain

$$\frac{1}{x_i}-\frac{1}{x_o}=\left(\frac{n_t-n_i}{n_i}\right)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

which for incident rays in air simplifies to the **Thin-lens equation** or **lensmakers formula** (signs depend on conventions used!)

$$\boxed{\frac{1}{x_i}-\frac{1}{x_o}=(n_t-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

As $x_i \to \infty$ we obtain the **first focal length** or **object focal length**

$$-\frac{1}{x_o}=(n_t-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=-\frac{1}{f_o}.$$

As $x_o \to \infty$ we obtain the **second focal length** or **image focal length**

$$\frac{1}{x_i}=(n_t-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{1}{f_i}.$$

Hence we obtain the **Gaussian lens formula**

$$\boxed{\frac{1}{x_i}-\frac{1}{x_o}=\frac{1}{f_i}}$$

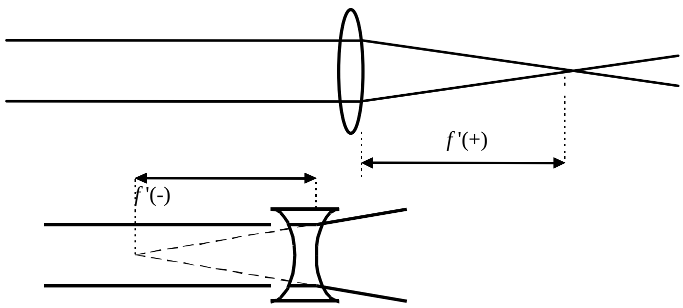
For thin lenses $f_o=-f_i$ in our convention and so

$$\boxed{\frac{f_i}{x_i}+\frac{f_o}{x_o}=1}$$

## Lens Power

The power of a lens, *P*, is usually expressed in dioptres, which is just the inverse of the focal length in metres:

$$\text{lens power in dioptres} = \frac{1}{\text{focal length in metres}} .$$




A *converging* lens is said to have a *positive* power and a *diverging* lens is said to have a *negative* power

## Vergence

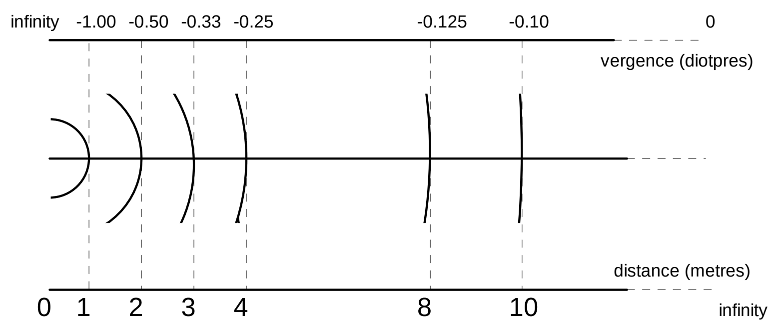
The amount by which light rays converge or diverge is known as their *vergence.* The vergence of light in a vacuum (or in air) is the same as the radius of curvature of the wavefronts. In a medium with refractive index *n*, the vergence is:

$$V = n \,/\, \text{wavefront curvature } (n \sim 1 \text{ for air})$$

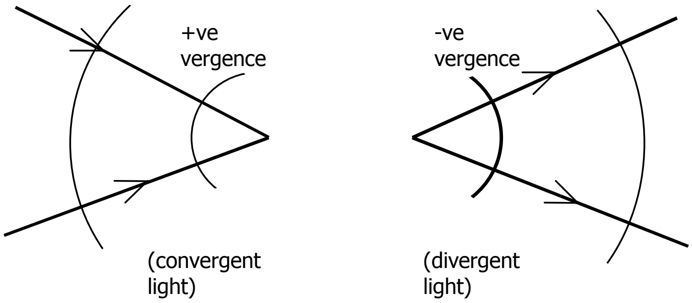
At distance, *d*, from a point source of light, the vergence of light (in air) is

$$V = \frac{1}{d}$$

Like the lens power, vergence can be expressed in dioptres.



In order to distinguish between converging and diverging light, a positive value is given to converging light and a negative value is given to diverging light.

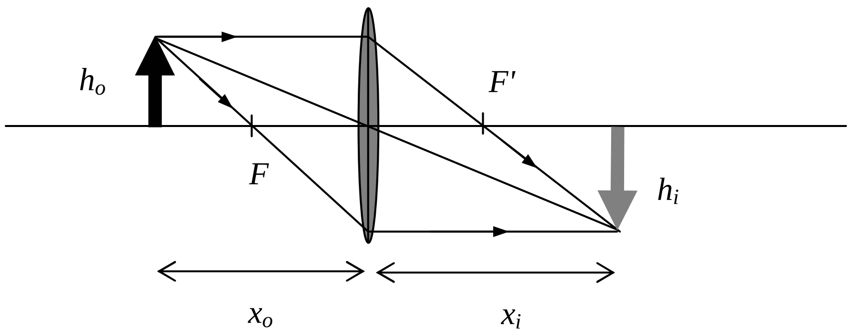



Vergences can be used in the lens formulae, but care must always be taken to ensure that the correct vergence is used at each point of the calculation.

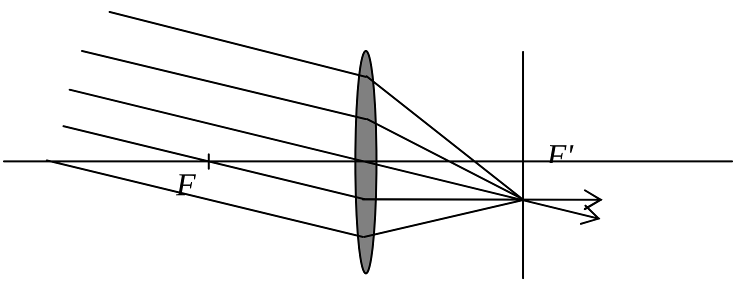
## Ray tracing Rules for thin lenses

Thin lenses behave in an ideal way and some rays are easily drawn:

1. A ray parallel to the axis is deflected through F or as if it came from F.
2. A ray through the optical centre of the lens continues undeviated
3. A ray to the lens that (when extended, if necessary) passes through F is deflected parallel to the axis.




An additional useful rule is
4. Rays parallel to each other are imaged on the focal plane



## Magnification

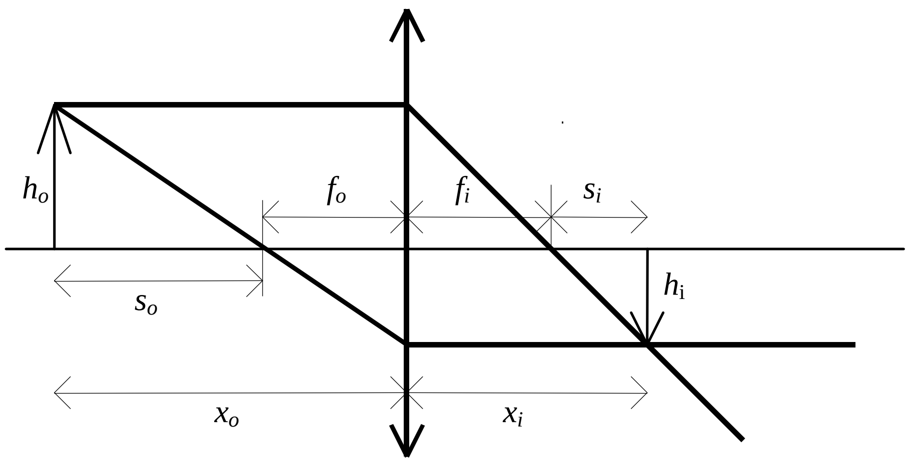
As with spherical mirrors transverse ($M_T$) and longitudinal ($M_L$) magnifications can be defined. By similar triangles

$$M_T = \frac{h_i}{h_o} = \frac{x_i}{x_o} = M_L \ .$$

Using our convention $M_T$ is negative indicating an inverted image. (Note that if the refractive indices on opposite side of the lens were different this would not be true.)

## Newton's Relation

Since the foci of a thin lens in air are symmetrical with respect to the lens, the magnification can be expressed in terms of the extra-focal distance, $s_o$, $s_i$ (that is the distance of the object or the image to the focal point). The following diagram explains this:




By similar triangles:

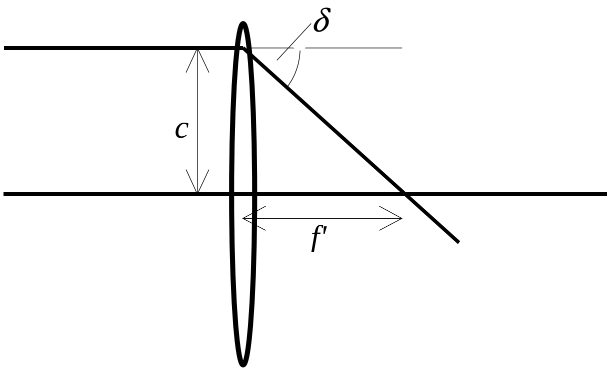
$$\frac{h_o}{f_i} = \frac{h_i}{s_i} \Rightarrow M_T = \frac{s_i}{f_i} \quad \text{and} \quad \frac{h_i}{f_o} = \frac{h_o}{s_o} \Rightarrow M_T = \frac{f_o}{s_o}$$

Hence $\boxed{s_o s_i = f_o f_i}$

which is Newton's relationship. Newton's relationship is also true for thick lenses.

## Prismatic effect of a thin lens

A thin lens can be considered to be made up of an infinite number of prisms. For a convex lens, these would be prisms with their bases towards the optical centre of the lens (see diagram below). If a ray strikes the lens a distance *c* above the axis, and passes through the focus of the lens, it is deviated through an angle $\delta$.




Since $\tan\delta = \dfrac{c}{f'}$

and since, for a prism, $100\tan\delta = \Delta$ (prismatic power in dioptres)

$$\text{then}\quad \Delta = \frac{c}{f'} \quad \text{(if } c \text{ is in cm and } f' \text{ is in metres)}$$

Therefore, $\Delta = cP$ where $\Delta$ = prism power in dioptres
$c$ = decentration in cm
$P$ = lens power in dioptres

This is known as the **Prentice rule**, and is useful for determining the prismatic effect induced by decentring a lens.