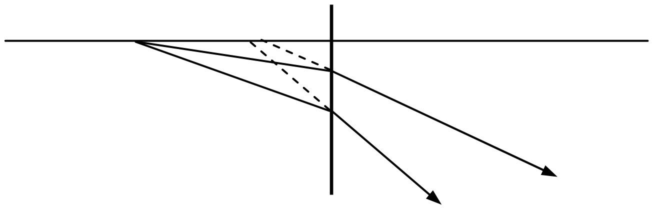
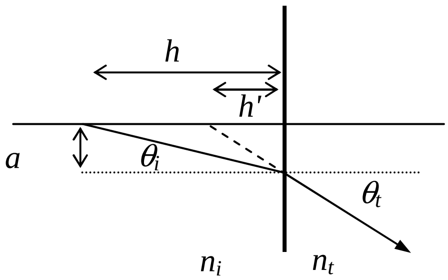
Optics 6 Geometrical Optics: Refraction

Refraction at a single surface

Refraction of a point source by a plane surface does not result in a point image - the image position depends on the angle of incidence.



Calculation of image position (apparent depth)

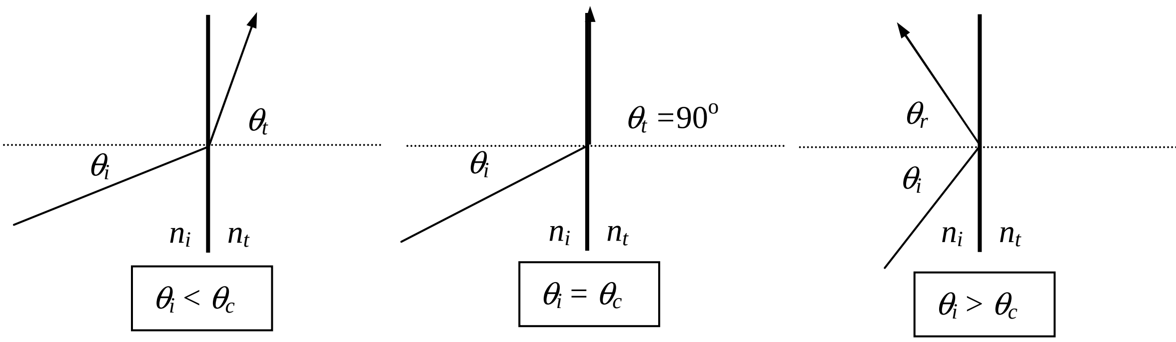


$$\frac{a}{h} = \tan\theta_i \quad ; \quad \frac{a}{h'} = \tan\theta_t$$

$$\frac{h'}{h} = \frac{\tan\theta_i}{\tan\theta_t} \approx \frac{\sin\theta_i}{\sin\theta_t} \text{ for small } \theta_i,\ \theta_t . \qquad \text{Hence } \boxed{\frac{h'}{h} \approx \frac{n_t}{n_i}}.$$

E.g. for water n_i = 1.33, air n_t = 1.0 $\Rightarrow h'/h \approx 0.75$ when viewed from above.

Total Internal Reflection



If $n_t > n_i$ and if θ_i becomes sufficiently large θ_t reaches 90° and sin θ_t = 1. This condition defines the critical angle θ_c

$$\sin\theta_c = \sin\theta_i = \frac{n_i}{n_t}$$

- If $\theta_i < \theta_c$ light is refracted and some is reflected at the interface.
- If $\theta_i > \theta_c$ total internal reflection occurs and $\theta_r = \theta_i$.

Uses of total internal reflection:

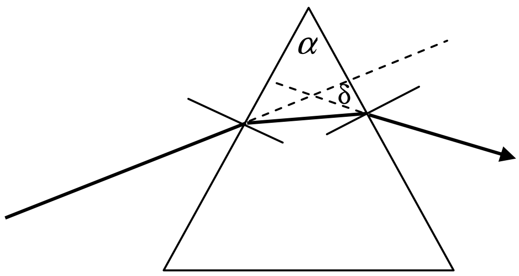
- prisms may be used as reflectors in certain optical instruments
- fibre optics (fibrescopes)
- measurement of refractive index

Prisms

Prisms and their combinations serve many purposes in optics and have many shapes, but most make use of one of two main properties.

- Dispersing prisms - maximise dispersion.
- Reflecting prisms - use total internal reflection to minimise dispersion.

However, ophthalmic prisms are used primarily to deviate an incoming beam.



Prism nomenclature:

- **refracting angle, α, (prism angle, apical angle)** Effect of prism also depends on refractive index

- **angle of deviation, δ** From above,
  Approximation for ophthalmic prisms relative refractive index n_r:

$$\delta = (n_r - 1)\alpha$$
$$\therefore \text{ for glass, } \delta \approx 0.5\alpha$$

- **Centrad: ∇ –** deviation is measured on a circular arc, such that a prism of power *x* centrads produces a deviation of *x* cm on an arc of radius 1 metre.

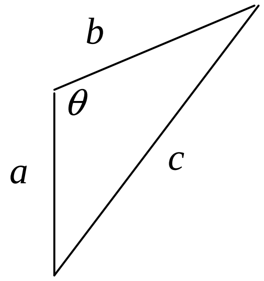
- **Prism dioptre: Δ –** deviation is measured in terms of apparent displacement, such that a prism of *x* dioptres produces an apparent displacement of *x* cm on a flat screen at distance of 1 metre. 1 prism dioptre causes a deviation of 1 in 100.

Effect of prism on visual direction

A prism deviates light in the direction of the base. Therefore, *objects appear shifted towards the apex of a prism.*

Addition and combination of prisms

Often we need to know the effect of two prisms which are combined in some way. There are two simple methods for working out the resultant prismatic power when two prisms are put together:



- Use the cosine rule and geometry

$$c^2 = a^2 + b^2 - 2ab\cos\theta$$

- Draw a graph

Fresnel prisms

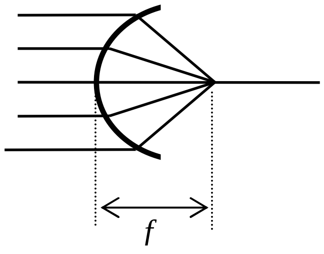
The Fresnel prism reduces the substance of a prism, but still gives the prismatic effect.

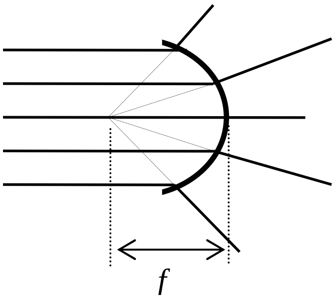
Pros	**Cons**
lighter - more comfortable	difficult to keep clean
cosmetically superior due to reduced thickness	optically inferior
should be cheap to manufacture	durability questionable
convenient	
high powers possible	

Refraction at spherical surfaces

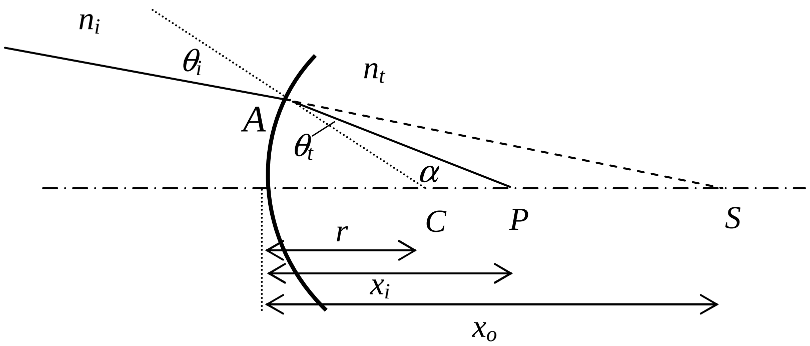
A convex spherical refracting surface will cause parallel rays of light to be converged to a point. This point is known as the *focus* or *focal point*, and its distance from the refracting surface is known as the *focal length f.*

A concave refracting surface causes parallel rays of light to diverge, and the focal point is the point through which the rays appear to have passed. In this case, the focal length is a negative distance (from our sign convention).





Gaussian Optics The figure below represents a spherical surface with rays shown for $n_t > n_i$; e.g. with air on the left and glass on the right.



Following reasoning similar to that used for the spherical mirror, by the sine rule:

$$\frac{SC}{\sin(\theta_i)} = \frac{SA}{\sin(\alpha)} \Rightarrow \frac{SC}{SA} = \frac{\sin(\theta_i)}{\sin(\alpha)}$$

and $$\frac{CP}{\sin(\theta_t)} = \frac{AP}{\sin(\alpha)} \Rightarrow \frac{CP}{AP} = \frac{\sin(\theta_t)}{\sin(\alpha)}$$

But here θ_i and θ_t are not usually equal. In the paraxial approximation we can put $SA \approx x_o$ and $AP \approx x_i$ then using

$$SC = x_o - r \ , \ CP = x_i - r$$

$$\frac{SC}{SA} \approx \frac{x_o - r}{x_o} \quad \text{and} \quad \frac{CP}{AP} \approx \frac{x_i - r}{x_i}$$

Taking the ratio of these and introducing Snell's law

$$\frac{SC/SA}{CP/AP} \approx \frac{x_o - r}{x_o}\frac{x_i}{x_i - r} = \frac{\sin\theta_i}{\sin\theta_t} = \frac{n_t}{n_i} \quad ,$$

giving $$\frac{x_o - r}{x_o} n_i = \frac{x_i - r}{x_i} n_t \quad ,$$

which re-arranges to the basic equation of Gaussian (paraxial) Optics

$$\boxed{\frac{n_i}{x_o} - \frac{n_t}{x_i} = \frac{n_i - n_t}{r}}$$

In the limit that $x_i \to \infty$ rays from a source at x_o are parallel to the axis and

$$f_o = x_o = \frac{rn_i}{(n_t - n_i)}$$ is the first or object focal length.

In the limit that $x_o \to \infty$ the incident rays are parallel to the axis and

$$f_i = x_i = \frac{rn_t}{(n_t - n_i)}$$ is the second or image focal length.