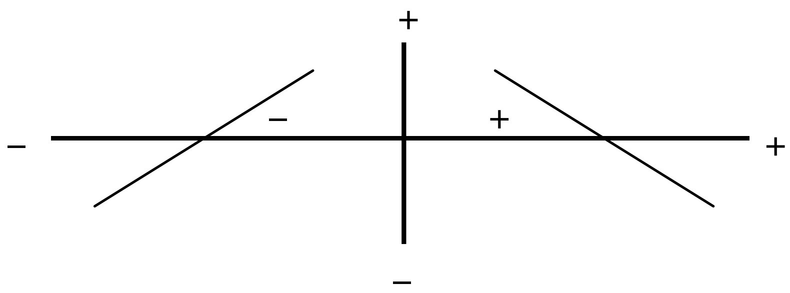
Optics 5 Geometrical Optics: Mirrors

Geometrical Optics is an idealised optics that essentially ignores the wave nature of light. It is optics for $\lambda \to 0$ in comparison to the objects encountered. It neglects interference, diffraction effects & polarization and uses rays to trace the path of light through reflecting and refracting bodies.

Optical Sign Convention

Several different conventions are commonly used - we will use the following sign convention where possible:

- figures are drawn with light traveling from left to right
- distances to the left of the vertex (eg lens surface) are *negative*
- distances to the right of the vertex are *positive*
- distances above the axis are *positive*
- distances below the axis are *negative*
- a ray that could be aligned with the principal axis by an *anticlockwise* rotation makes a *positive* angle with the axis
- a ray that could be aligned with the principal axis after a *clockwise* rotation makes a *negative* angle with the axis



Fermat's principle

Fermat (1601-1655) pointed out that nature is economical, and that light will therefore travel along the path that takes the least time. This is quite a good rule but there are some instances where it is erroneous and the modern statement is extended say that *the ray path along which light travels is such that the time taken is a stationary value* (ie minimum, maximum or constant) Fermat's principle can be used to *geometrically* derive the laws of reflection and refraction.

Optical Path Length

One of the most important principles in geometrical optics is that of optical path length (OPL) - this is the *distance* that light would have travelled if it were in a *vacuum* instead of some optical media for the same amount of time. An alternative statement of Fermat's principle is that light traverses an OPL that is stationary with respect to variations of that path.

Why Lenses and Mirrors Focus Light

A focusing (converging) lens is thicker at the center so light rays passing through the center take longer to travel through the lens than light-rays on the other parts. By

Fermat's Principle, all light-rays from the source to the objects have the same optical path length. Although the path along the optical (or principle) axis has the shortest geometric distance, it also has to travel through the greatest thickness of glass. From a wavefront view-point, the expanding wavefronts from the source become contracting wavefronts that converge on the image plane.

A focussing mirror has a concave curved surface. Again, it reflects the expanding wavefront back so that it becomes a converging wavefront. Alternatively, the geometric paths lengths are the same for all rays between source and object.

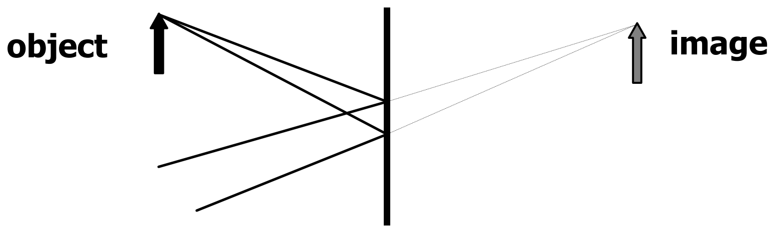
Lenses and mirrors can also be made that *diverge* light-rays. Lenses of this type are thinner at the center. Mirrors of this type have a convex reflecting surface.

Real and Virtual Images

Real images are formed when the light-rays from an object actually cross. If a screen is placed at this point, an image will be formed on it. A *virtual image* is formed when the light-rays appear to come from a point although they never actually cross at this point and so the image cannot be formed on a screen.

Image formation by reflection at a plane surface

From geometry, the reflection of light from a point source by a plane surface forms a virtual image of the source.



Focal points and focal length

A *convex* spherical refracting surface or a *concave* spherical reflecting surface will cause *parallel rays* of light to be converged to a point. This point is known as the *focus* or *focal point*, and its distance from the optical surface is known as the *focal length*. For divergent lenses and mirrors the focal point is the virtual image point through which the rays appear to have passed. In this case, the focal length is a negative distance (from our sign convention).

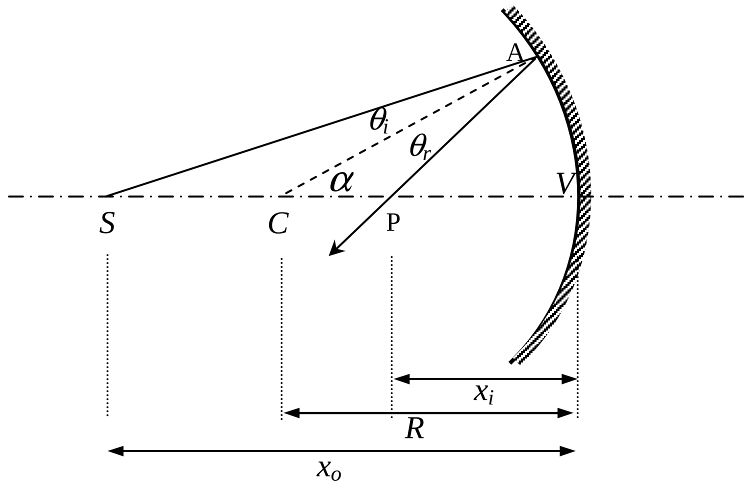
The Paraxial Approximation

To start with, we will only consider lenses and mirrors in the *paraxial approximation*. This means that we only consider light-rays that travel *close to the optical axes*. This considerably simplifies the equations.

It is also much easier in practice to construct surfaces with a spherical profile, than it is to produce e.g. parabolic surfaces.

Mirror formula

In the figure light from a source S is reflected off the spherical concave mirror radius R centered on C.



By the sine rule (or you can use geometry)

$$\frac{SC}{\sin(\theta_i)} = \frac{SA}{\sin(\pi-\alpha)} = \frac{SA}{\sin(\alpha)} \Rightarrow \frac{SC}{SA} = \frac{\sin(\theta_i)}{\sin(\alpha)}$$

and $$\frac{CP}{\sin(\theta_r)} = \frac{AP}{\sin(\alpha)} \Rightarrow \frac{CP}{AP} = \frac{\sin(\theta_r)}{\sin(\alpha)}$$

By the law of reflection θr and θi are equal, hence $\frac{SC}{SA} = \frac{CP}{AP}$

In the paraxial region (close to centre line) $SA \approx x_o$ and $AP \approx x_i$

So $$\frac{x_o - R}{x_o} = \frac{R - x_i}{x_i}$$

giving the mirror formula $$\boxed{\frac{1}{x_o} + \frac{1}{x_i} = \frac{2}{R}}$$

Primary or Object focus $\lim_{s_i \to \infty} x_o = f_o$, Secondary or image focus $\lim_{s_o \to \infty} x_i = f_i$

$f_o = f_i = \frac{1}{2}R$ dropping subscripts $\frac{1}{x_o} + \frac{1}{x_i} = \frac{1}{f}$

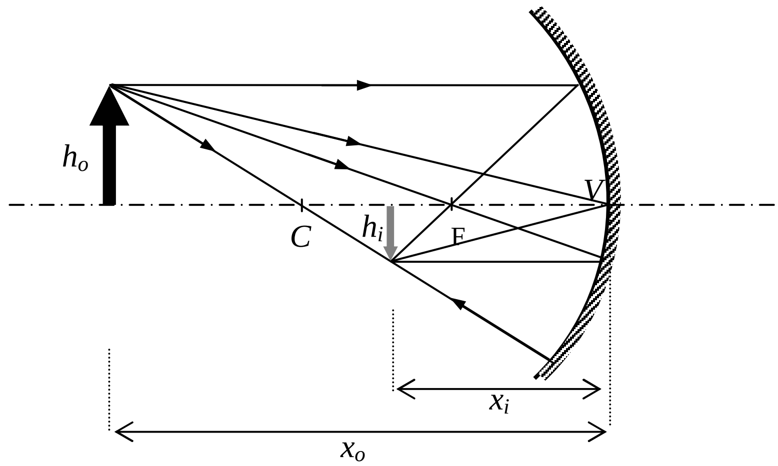
These formulae also apply to convex mirrors but note that *R* is negative.

Linear Magnification: ray tracing

Some rays are easy to draw and with an accurate diagram we can construct the image position.

1. Rays (when extended) that pass through C are reflected back on themselves

2. Rays (or rays with an extension) that pass through F are reflected back parallel to the axis
3. Incident rays parallel to the axis are reflected back through F (or have a reflection with an extension that passes through F).
4. Recall also that $\theta_i = \theta_r$ and that this imposes a symmetry on rays that strike V.



Any two of these rays are sufficient to define the image position.

When an extended object is used, the image is often of a different size to the object, and magnification has taken place. Transverse and longitudinal magnification are both examples of linear magnification.

Transverse magnification (M_T) is the ratio of the *heights* of the image and object. From the figure for a spherical mirror,

$\frac{h_o}{x_o} = -\frac{h_i}{x_i}$ by similar triangles. Then the transverse magnification is

$$M_T = \frac{h_i}{h_o} = -\frac{x_i}{x_o}$$

Negative indicates an inverted image.

Longitudinal magnification (M_L) is the ratio of the *lengths* (along the axis) of the image and the object, and although it is not often used in ophthalmology it can be of great significance in photography.)

$$M_L = \frac{x_i}{x_o}$$

Note that for a plane mirror $M_T = -1$, $M_L = +1$.